Duration Dependence And Mean Reversion: An Attempt Of Identification In Tunisian Stock Market

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ABSTRACT

This study investigates the duration dependence of Tunisian stock market over the period from January 07, 1998 to March 29, 2013; using two-state Duration Markov-switching model. Through this model, duration dependence is emphasized in the conditional mean return, volatility, risk-return trade-off and transition probabilities. We demonstrate that TUNINDEX index weekly returns can be sorted into bull and bear market states. Our results are consistent with mean reversion process; i.e. mean reversion is the tendency of asset prices to return to a trend path. Finally, we conclude that Tunisian stock market fluctuations can be characterized by the presence of the countercyclical return volatility due to the asymmetric movements of the risk premia.

Keywords*:* Regime-Switching; Duration-Dependence; Mean-Reversion Process; Countercyclical Return Volatility

1. INTRODUCTION

Several researchers in finance have long been interested in the long-run time-series properties of equity prices, with particular attention to if stock prices have undergone cycles of bull and bear phases. This study lies in the same perspective and attempts to investigate many issues peculiar to the stock market cycles. The primary issue we study is whether the periodicity in market cycles may exist. The presence of periodicity of stock market cycles has irrelevant implications for stock prices behavior. As a first step, as a cycle extends, the probability that it terminates should raises. As a second step, the presence of periodicity of market cycles suggests that the length of stock market cycle could be used to predict potential turning points in the stock market cycle. In this regard, the classical depiction of stock prices as random walk process seems to be unsuitable with the periodicity of stock market cycle. Random walk process suggests that any shock to stock price is permanent and there is no tendency for the price level to turn up to a trend path over time. As a result, investors are unable to forecast future returns based on lagged prices. As a third step, the tendency for stock prices to cyclical pattern involves that the stock market is suitable with the mean reversion in long-horizon stock prices as indicated in many studies (e.g. Poterba and Summers, 1988; Fama and French, 1988; Cochran and DeFina, 1995).

From a statistical standpoint, the question of whether stock prices have a tendency towards cyclical pattern may be bordered as one of duration dependence. The key issue to be investigated is if the probability that stock market cycle will end grows as the stock market cycle extends. If this is the case, then stock prices can be characterized as mean-reverting process. Early studies have beforehand analyzed the issue of mean reversion in financial time-series; see, for example, Fama and French (1988), Poterba and Summers (1998), Lo and Mackinely (1988), Kim et *al*. (1991), Richardson (1993), Harvey (1995), Balves, Wu et Gilliland (2000), Kim, Stern et Stern (2009), Gil-Alana (2007), Cunado, Gil-Alana et de Gardia (2010), Cassano (1990); Hillebrand (2005), Chaudhuri and Wu (2003), among others. Despite of these extensive studies, nevertheless, researches report conflicting empirical evidence on mean reversion. DeBondt and Thaler (1985) show that past losing stocks over the past 3-5 years significantly outperform the previous winning during the same period. Their results indicate that stock returns tend to be mean reverting. Fama and French (1988) also report that U.S. stock prices exhibit mean reversion over long horizon. Richands (1997) documents evidence of long-term winner-loser reversals for equity indexes for 16 countries. Balves, Wu and Gilliland (2000) provide strong evidence in 18 developed equity markets. Besides, they show that the time-series property (i.e. mean reversion) cans be exploited in order to predict equity returns by using a parametric contrarian investment strategy. Recently, Chaudhuri and Wu (2003) identify the mean reverting component in equity prices across 17 emerging markets. Yet, other researches do not support the mean reversion in stock prices (e.g. Lo and Mackinlay, 1988; Richardson and Stock, 1989). For instance, Lo and Mackinlay (1988) report evidence against mean reversion in weekly U.S. stock prices. Kim, Nelson et Startz (1991) prove that mean reversion exists only in prewar U.S. data. Richardson and Stock (1993) argue that results from Poterba and Summers (1988) and Fana and French (1988) are not robust due to small-sample bias problems. Lo (1991) finds no evidence against the random walk hypothesis. Cunado, Gil-Alana and Gracia (2010) find little or no evidence of mean reversion in the U.S. stock market prices during the period 1929-2006.

Much controversy related to the mean reversion process arises on the grounds that the speed of reversion, if it exists, may be slow (Chaudhuri and Wu, 2003). In this regard, Balves, Wu and Gilliland (2000) argue that the detection of the mean reversion is so difficult because of the need to identify a trend path (or fundamental value) for the stock prices. Summers (1996) reports that available econometric methods generally lack the power to discriminate between mean reversion and random walk processes. Early studies on mean reversion in the stock prices used variance ratio tests (e.g. Poterba and Summers, 1988; Cecchetti et al., 1990; Lo and Mackinlay, 1988); traditional unit root tests such as Augmented Dickey Fuller (ADF), Phillips and Perron (PP); or some recent developments on the basis of these tests (e.g. Eliott et al., 1996; Ng and Perron, 2001). Other recent studies employ unit root test which allow for one or more structural breaks (e.g. Phengpis, 2006; Narayan and Smyth, 2007); or panel data tests (e.g. Choi and Chue, 2007; Chaudhuri and Wu, 2003). Another line of research is based on the long memory techniques (e.g. Caporale and Gil-Alana, 2004; Assaf, 2006) and the possibility of non-linear behavior in the mean reversion process (e.g. Bali et al., 2008; Lim and Liew, 2007).

This study attempts to investigate duration dependence in the Tunisian stock market cycle during the period from January 07, 1998 to March 29, 2013 in order to examine the issue of mean reversion. Mean reversion is defined as the change of index return towards a reversion level as a reaction to previous change in index return. Following a positive (resp. negative) variation in the actual returns, mean reversion induces a negative (resp. positive) successive change. Methodologically, we use the two-state Duration-Dependence Markov-switching model in which duration dependence is emphasized in the conditional mean return, volatility, risk-return trade-off as well as the transition probabilities. In particular, we investigate duration-dependent transition probabilities using parametric hazard functions. Hazard functions are well-adapted in order to study duration dependence since they specify the probability that a particular state will terminate conditional on the time which has been spent in this state.

The outline of the paper is as follows. Section 1 exhibits the model development. The financial dataset used for empirical application is presented in section 2. Section 3 reports estimation results for two-state Markov-switching model applied to TUNINDEX index returns and provides features of different states. Finally, section 4 is devoted to conclusions and final remarks.

2. MODEL DEVELOPMENT

In order to examine the mean reversion in stock returns, we adopt a two-state Markov-switching model which specifies parsimoniously the duration dependence simultaneously in conditional mean of returns, volatility, risk-return trade-off and the transition probabilities. Especially, We use a two-state Duration-Dependence Markov-Switching L^{th} order Autoregressive Duration-Dependence GARCH-M (DD(τ)-MS(2)-AR(L)-DD(τ)-GARCH-M(1,1)) model.

At each time t, the return series is assumed to belong to one of two states. Let S_t^* denotes a latent variable which takes the values {1, 2}. The transition dynamics between the two states is depicted by a homogeneous semi-Markov process. The transition intensities depend on duration which expressed by a latent variable$\mathrm{D}_{S_t^*}$.$\mathrm{D}_{S_t^*}$is the number of successive periods recently spent in the same regime:

$$D_{S_t^*} = \begin{cases} D_{S_{t-1}^*} + 1 \text{ if } S_t^* = S_{t-1}^* \\ 1 \text{ otherwise} \end{cases} \tag{1.1}$$

Like Maheu and McCurdy (2000), the transition probabilities are parameterized using the logistic function. For each state *i* (*i* = 1, 2), the transition probabilities are specified conditionally on $D_{S_{t-1}^*}$ $D_{S_{t-1}^*}$ as follows:

$$p_{ij}^{d} = \Pr\left(S_t^* = j / S_{t-1}^* = i ; D_{S_{t-1}^*} = d\right) = \frac{\exp\left[\lambda_1^{ij} + \lambda_2^{ij}\left(d\, I_{(d\le\tau)} + \tau\, I_{(d>\tau)}\right)\right]}{1+\exp\left[\lambda_1^{ij} + \lambda_2^{ij}\left(d\, I_{(d\le\tau)} + \tau\, I_{(d>\tau)}\right)\right] + \exp\left[\lambda_1^{ik} + \lambda_2^{ik}\left(d\, I_{(d\le\tau)} + \tau\, I_{(d>\tau)}\right)\right]}^{1} \tag{1.2}$$

From equation (1.2), the probability p_{ij}^d refers to hazard functions which reflect the instantaneous probability to change from the states *i* to *j* given that the state *i* has spent *d* periods. Nevertheless, it is possible that there is no regime change during other periods indicating the persistence in state i:

$$p_{ii}^{d}\left(S_t^* = i / S_{t-1}^* = i ; D_{S_{t-1}^*} = d\right) = 1 - p_{ij}^{d} \tag{1.3}$$

The effect of duration on the hazard functions (or transition probabilities) is only summarized by the coefficients λ_2^{ij}. $\lambda_2^{ij} > 0$ (resp. < 0) implies positive (resp. negative) duration dependence. In this respect, the hazard functions are increasing (resp. decreasing) depending on the age of state *i*. Moreover, the level of persistence in state i is decreasing (resp. increasing) with duration. By assumption, this level becomes constant beyond the horizon of τ periods. According to Durland and McCurdy (1994) and Maheu and McCurdy (2000), the parameter τ refers to the memory[2] of duration dependence.

Formally, our DD(τ)-MS(2)-AR(L)-DD(τ)-GARCH-M(1.1) model[3] (henceforth, two-state DD-MS-GARCH-M model) may be written as:

$$R_t(S_t) = \alpha^{(S_t^*)} + \delta^{(S_t^*)} \ln\left(d_{S_t^*}\right) + \sum_{l=1}^{L} \beta_l^{(S_{t-l}^*)}\left[R_{t-1} - \alpha^{(S_{t-l}^*)} - \delta^{(S_{t-1}^*)} \ln\left(d_{S_{t-1}^*}\right)\right] + \eta^{(S_t^*)} \ln\left(d_{S_t^*}\right) h_t(S_t) + \varepsilon_t \tag{1.4}$$

$$h_t(S_t) = \gamma_0^{(S_t^*)} e^{\gamma_1^{(S_t^*)} \ln\left(d_{S_t^*}\right)} + \gamma_2^{(S_{t-1}^*)} \left(\tilde{\varepsilon}_{t-1}^{(S_{t-1}^*)}\right)^2 + \gamma_3 \tilde{h}_{t-1}, \tag{1.5}$$

$$\tilde{\varepsilon}_{t-1}^{(S_{t-1}^*)} = R_{t-1} - E_{t-2}\left[R_{t-1} / S_{t-1}^*\right], \tag{1.6}$$

$$E_{t-2}\left[R_{t-1} / S_{t-1}^*\right] = \frac{\sum_{n=1}^{N} E_{t-2}\left[R_{t-1} / S_{t-1} = n, Y_{t-2}\right] \Pr\left(S_{t-1} = n / Y_{t-1}\right) I_{(S_{t-1}^*)}}{\sum_{n=1}^{N} \Pr\left(S_{t-1} = n / Y_{t-1}\right) I_{(S_{t-1}^*)}}, \tag{1.7}$$

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[1] $I_{(d\le\tau)}$ and $I_{(d>\tau)}$ refer two dummy variables: $I_{(d\le\tau)} = \begin{cases} 1 & \text{if } d\le\tau \\ 0 & \text{otherwise} \end{cases}$ and $I_{(d>\tau)} = \begin{cases} 1 & \text{if } d>\tau \\ 0 & \text{otherwise} \end{cases}$

[2] Given that the parameter τ can take only discrete value, it is chosen (i.e. grid search method) to maximize the log-likelihood function starting from $\tau_{\min} = l + 1$.

[3] A statistical approach is used in order to justify the relevance of adopted specification. The series of TUNINDEX index weekly returns was tested for stationarity by applying Augmented Dickey-Fuller and Phillips-Perron test. The actual values of the autocorrelations and partial autocorrelations and the Ljung-Box test results for filtered weekly series (i.e. the series of residuals provided by adjusting the series of returns by ARIMA model) are used. Finally, the Lagrange multiplier test is established to analyze ARCH effect in the filtered series.

$$\tilde{h}^2_{t-1} = \sum_{n=1}^{N} h^2_{t-1}(n)\, \Pr(S_{t-1} = n / Y_{t-1}) \quad , \tag{1.8}$$

and

$$\varepsilon_t \to NID(0, h(S_t)),$$

where:

- R_t: Market index return at time t;
- Y_t : $(R_t, R_{t-1}, \ldots, R_1)$, the market index return's vector for periods t, t-1, ..., 1;
- S_t: {1, 2, 3, ..., N}, a "N states" latent variable, according to a first-order Markov chain characterized by the following property :
- $p(S_t = n | S_{t-1} = m, \ldots, Y_{t-1}) = p(S_t = n | S_{t-1} = m, Y_{t-1}) = p_{nm}$,n, m = {1, 2, 3, …, N} ; (1.9)
- $p(S_t = n / Y_{t-1})$, n = {1, 2,…, N}, smoothed probabilities' vector, which expressed, for any time t, the unconditional probabilities of states occurrence, knowing all the information up to time t-1.

Each realization of the variable S_t indicates the possible (L + τ) length trajectory of two-state Markov-Switching process. By construction, the path is identified to a vector of weekly situations depicting the $S_t^*$and$D_{S_t^*}$ and $D_{S_t^*}$ variables dynamics over the temporal spells of (L + τ) periods (week). The variable $I_{(S_t^*)}$ denotes the paths achieving to the same situation S_t^* at the end of spell.

From equation 1.4, the market index return' evolution is governed by an *L* order autoregressive process. Besides, the Ljung-Box (1978) test is applied to the standardized residuals in order to pronounce on the L autoregressive order. The error terms are unobserved because states are unobservable. According to Dueker (1997) and Maheu and McCurdy (2000), we use for this test the standardized expected residuals[4]:

$$\sum_{J=1}^{N} \frac{R_t - E[R_t / S_t = n, Y_{t-1}]}{\sqrt{h_t(S_t)}} \Pr(S_t = n / Y_{t-1}) \tag{1.10}$$

We also express the autoregressive terms of the volatility equation (eq. 1.5) in line with the $\tilde{\varepsilon}_{t-k}$ and $\tilde{h}_{t-1}^{(S_{t-1}^*)}$ terms rather than the ε_{t-k} and $h_{t-1}^{(S_{t-1})}$, respectively.

Equations (1.4) and (1.5) indicated that the conditional mean and variance can change with duration. This contributes to investigate dynamic behavior for the mean and the variance within each state. The exponential form used in equation (1.5) allows us to ensure the positivity of the conditional variance. From equation 1.4, We can elucidate the nature of the relationship between return and risk (i.e. risk-return trade-off) at each state through the link between the conditional mean and variance.

2. DATA

The data used for this study is based on a sample of The TUNINDEX index weekly returns calculated from the Tunisian Stock Exchange over a period[5] from 07/01/1998 to 29/03/2013. The total number of observations is 931. Table 1 reports the descriptive statistics of the data.

[4]Under these conditions, the Ljung-Box (1978) test results can be used as an indication because the asymptotic distribution of the statistics is unknown.

[5]The TUNINDEX index was launched in January 1998 by the authorities of Tunisian stock market.

Table 1. Descriptive Statistics Of The Tunindex Weekly Return Data

	07 Jan 1998-23 Mar 2013
Mean	0.1668
Median	0.0864
Minimum	-10.9427
Maximum	9.3009
Standard Deviation	1.38022
Skewness	-0.0749 (0.080)
Kurtosis	12.4102 (0.160)***
Jarque-Bera	5905.241
Probability	0.000000

Notes: - Summary statistics for TUNINDEX index returns from 07/01/1998 to 29/03/2013;
- Standard errors are displayed as (.); -***: Significance level at 1%.

As presented in table 1, the TUNINDEX index weekly returns vary between -10.9427% and 9.3009%. The mean weekly TUNINDEX return is 0.1668% and the return standard deviation is 1.3802%. The skewness and Kurtosis coefficients reveal departure from normality in the data, confirmed by the Jarque–Bera statistic. These descriptive statistics show high volatility which marks the TUNINDEX index weekly evolution. These values are also a reflection of strong stock price fluctuations on the Tunisian stock market. This apparent feature of the data set is illustrated in figure 1.

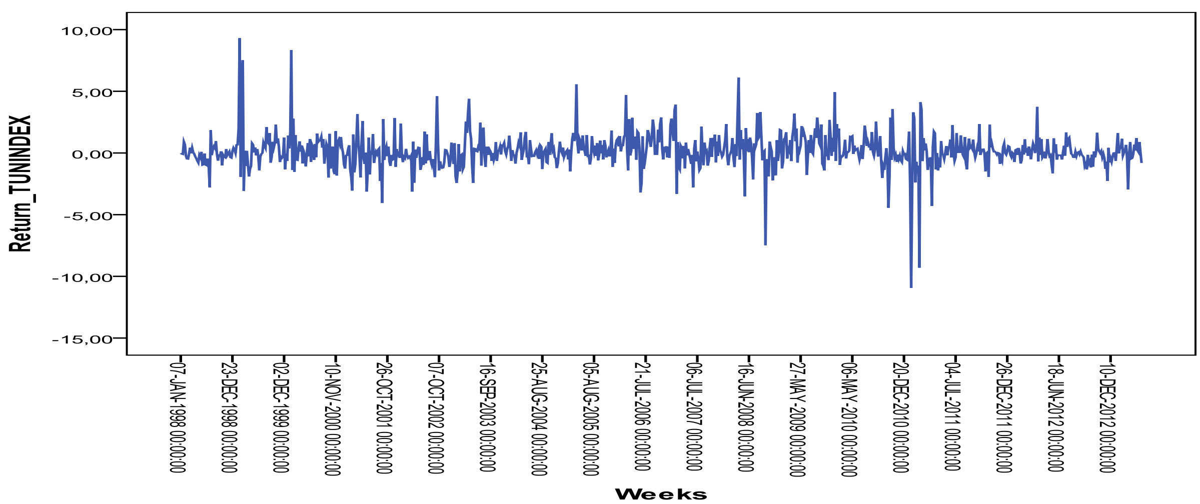

Figure 1. TUNINDEX Index Weekly Evolution (07/01/1998 to 29/03/2013)

As a result of the excess volatility, the return distribution for TUNINDEX index is characterized by a large number of extreme values. Figure 2 which reports the boxplot of weekly returns for our sample confirms this evidence.

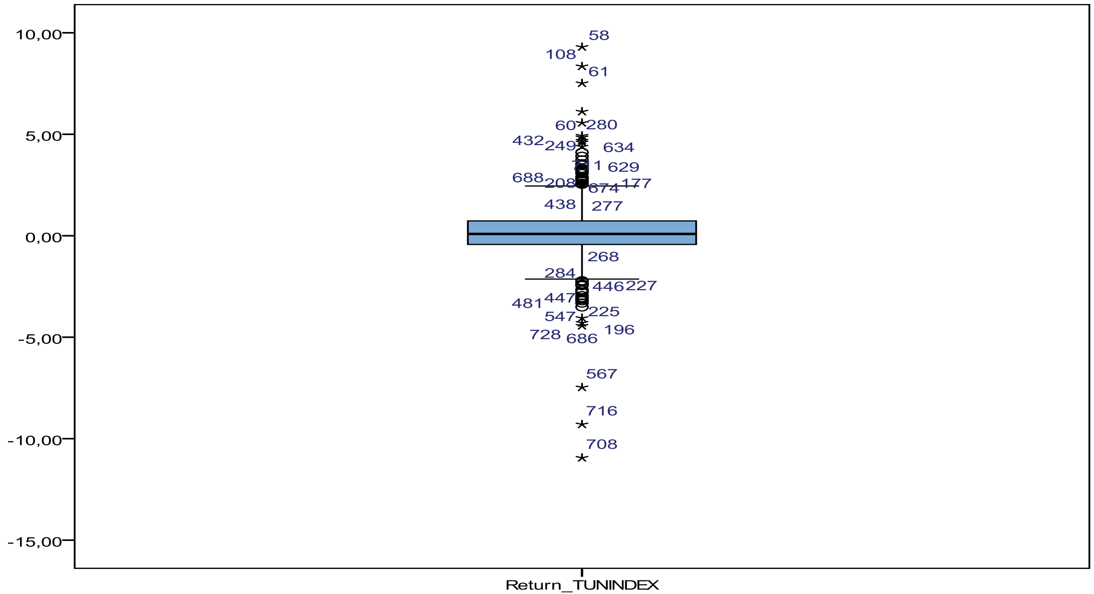

Figure 2. Boxplot

The linear dependence of stock returns is examined by applying the modified Ljung-Box Q statistic. Table 2 shows that the Q statistic is significantly different from zero for different lags. Thus, the presence of the linear autocorrelations in TUNINDEX index returns series is confirmed.

Table 2. Linear Autocorrelation Test Of Tunindex Index Returns

Qm(5)	Qm(10)	Qm(15)	Qm(20)	Qm(30)
40.077***	41.528***	48.188***	49.555***	63.830***

Notes: - Qm (k): modified Ljung-Box Q statistic; -***: Significance level at 1%.

To underline the existence of nonlinear dependence in the Tunisian index returns series, We use the BDS test to filtered data (i.e. the residual series estimated from ARMA model)[6]. As expected, the BDS statistic is significantly different from zero for different values of m and ε, implying the nonlinearity of the returns.

The value of Lagrange multiplier statistic of 362.209 is beyond 6.83 (the critical value of $\chi^2\chi^2$ (1) at the significance modeling volatility.

Table 3. BDS Test Statistic For The AR(2) Model Residual Series

M	ε/σ	BDS	ε/σ	BDS	ε/σ	BDS	ε/σ	BDS
2	0.5	33.80254***	1	35.29938***	1.5	34.39122***	2	33.52877***
3	0.5	36.02730***	1	34.70794***	1.5	32.82687***	2	30.72270***
4	0.5	38.76202***	1	33.74690***	1.5	30.94595***	2	28.23294***
5	0.5	42.36451***	1	33.07702***	1.5	29.26498***	2	26.26642***
6	0.5	47.08662***	1	32.79473***	1.5	27.96575***	2	24.76822***
7	0.5	53.39746***	1	32.84613***	1.5	26.88015***	2	23.48455***
8	0.5	61.41649***	1	33.12794***	1.5	26.01166***	2	22.39505***
9	0.5	71.98092***	1	33.59017***	1.5	25.32967***	2	21.46567***
10	0.5	85.39705***	1	34.19890***	1.5	24.76811***	2	20.73425***

Notes : m : the embedding dimension; ε : the proximity ; σ : residuals standard deviations ; *** Significance level at 1% ; the critical value of BDS test statistic is 2.58 at 1% level.

[6]The Box-Pierce test results display that we can limit to the second-order autoregressive model in order to estimate no-correlated residuals.

3. ESTIMATION RESULTS

The maximum likelihood estimates' results of two-state DD-MS-GARCH-M model are reported in table 4. The opposite signs of the conditional return' intercepts, $a^{(S_t^*)}$, display the existence of high-return and low-return states.

Table 4. Estimates Of Model (M2)

Parameters	*Model (M2) : DD(τ)-MS(2)-AR(2)-DD-GARCH-M (1,1)*
$\alpha^{(1)}$	2.3054 (0.0586) ***
$\alpha^{(2)}$	-0.8562 (0.0443) ***
$\beta_1^{(1)}$	-0.0557 (0.0145) ***
$\beta_1^{(2)}$	0.0620 (0.0151) ***
$\beta_2^{(1)}$	0.3549 (0.0064) ***
$\beta_2^{(2)}$	0.1464 (0.0086) ***
$\delta^{(1)}$	-0.2545 (0.0262) ***
$\delta^{(2)}$	0.1840 (0.0241) ***
$\eta^{(1)}$	0.0140 (0.0031) ***
$\eta^{(2)}$	-0.0111 (0.0054) **
$\gamma_0^{(1)}$	0.1959 (0.0110) ***
$\gamma_0^{(2)}$	0.1981 (0.0119) ***
$\gamma_1^{(1)}$	-1.2161 (0.0612) ***
$\gamma_1^{(2)}$	-1.7551 (0.0509) ***
$\gamma_2^{(1)}$	0.5722 (0.0370) ***
$\gamma_2^{(2)}$	0.2748 (0.0146) ***
γ_3	0.0530 (0.0060) ***
$\lambda_1^{(1,2)}$	-0.2282 (0.0950) ***
$\lambda_1^{(2,1)}$	-1.9232 (0.1304)***
$\lambda_2^{(1,2)}$	0.0831 (0.0141) ***
$\lambda_2^{(2,1)}$	-0.2590 (0.0526) ***
lgl	-787255462
N	931
τ	14
Tests	**Model (M2)**
Q(5)	4.231
Q(10)	9.012
Q(15)	13.543
Q(20)	20.012
Q(30)	35.654

Notes: -* Significant at the 10% level; - ** Significant at 5% level; - *** Significant at 1% level;
- Standard errors are displayed as (.); lgl is the log-likelihood value; N is number of observations

The two-state model also allows to show dynamic behavior of the conditional mean within each state. Indeed, the parameters that estimate the dependence of the conditional mean on duration, $\delta^{(1)}$ and $\delta^{(2)}$, are both significantly different from 0. During the first week in high-return state (resp. low-return state), the conditional return is about 2.3054 (resp. -0.8562) but if state 1 (resp. state 2) persists the conditional return declines (resp.

increases) over time. Therefore, state 2 delivers increasing low returns while state 1 is characterized by decreasing positive returns. Fig. 3.1 illustrates markedly the evolution of conditional return in states 1 and 2.

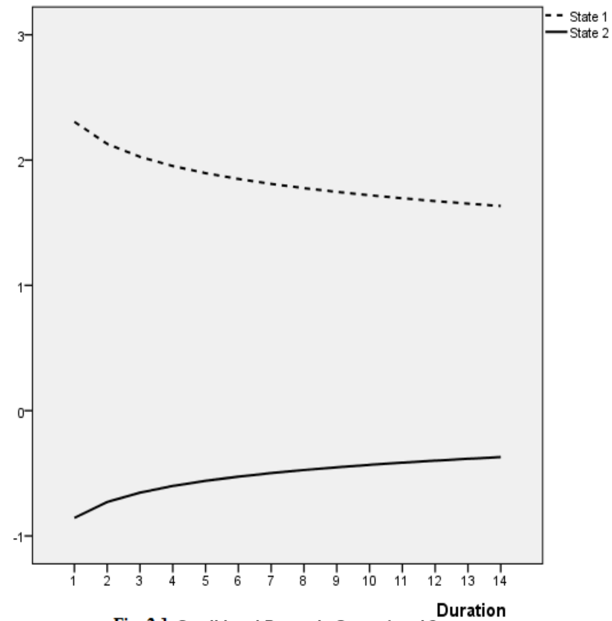

Fig. 3.1. Conditional Return in States 1 and 2

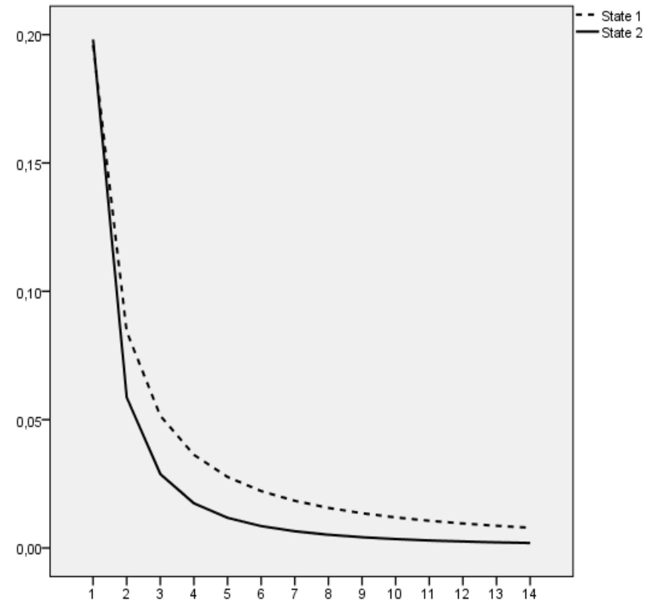

Fig. 3.2. Conditional Volatility in States 1 and 2

Figure 3. Conditional Return and Conditional Volatility From Two-State DD-MS-GARCH-M Model

In addition, the two-state model allows the conditional volatility to be dependent on duration. The coefficients, $\gamma_1^{(1)}$ and $\gamma_1^{(2)}$, which estimate this effect are both significantly negative. The difference between volatility in the two states is enough important ($\gamma_1^{(1)}$=-1.2161 and $\gamma_1^{(2)}$=-1.7551) to classify states into stable versus volatile. Nevertheless, Fig. 3.2 shows that the decline of the conditional volatility in state 1 is slightly higher than state 2 over time. This evidence could be related to the investors' changing risk preferences with the length of time spent in a particular market conditions. Figure 4 plots the evolution of the risk-return trade-off. The relationship between risk and return is neither stable over time nor linear. As pointed out by Lo (2004, 2005), if a relationship between risk and return exists, it tends unlikely to be stable over time. From figure 4, the difference between states in terms of risk-return trade-off is strongly pronounced. In fact, the relationship between risk and return in state 1 increases with duration while it declines in state 2. High returns in state 1 may be the lure which attracts investors to look for trading in the stock market. However, in low-return state, investors search to curb losses of their portfolio by using different strategies such as stop-loss.

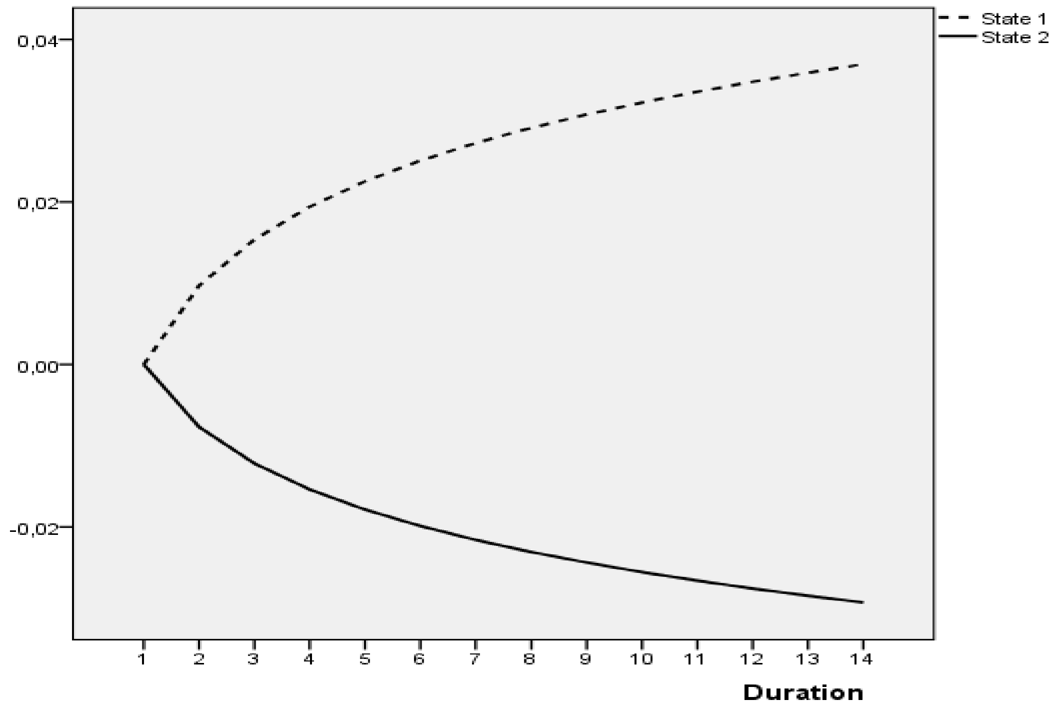

Figure 4. Risk-Return Trade-off in State 1 and 2

From table 4, the memory τ of duration dependence for the model was equal to 14. That is, duration is significant in impacting the transition probabilities for four months. The coefficients that outline the duration dependence in the transition probabilities, $\lambda_2^{(1,2)}$ and $\lambda_2^{(2,1)}$, are both significantly from 0. In particular, the coefficient $\lambda_2^{(1,2)}\lambda_2^{(1,2)}$ (resp. $\lambda_2^{(2,1)}$) is positive (resp. negative) implying an increasing hazard (resp. decreasing hazard). Figure 5 plots the transition and persistence probabilities over duration. The state 2 tends to be persistent as duration increases. However, the probability of staying in state 1 weakens over time. In this regard, the probability of exiting the state 1 (resp. the state 2) increases (resp. decreases) with duration. On average, the stock market spent 97% of time in the state 2 and only 3% in the state 1. These are the unconditional probabilities for states 1 and 2, i.e. $p(S=i)=\sum_{d=1}^{t} p(S=i,D=d), i=1,2$.

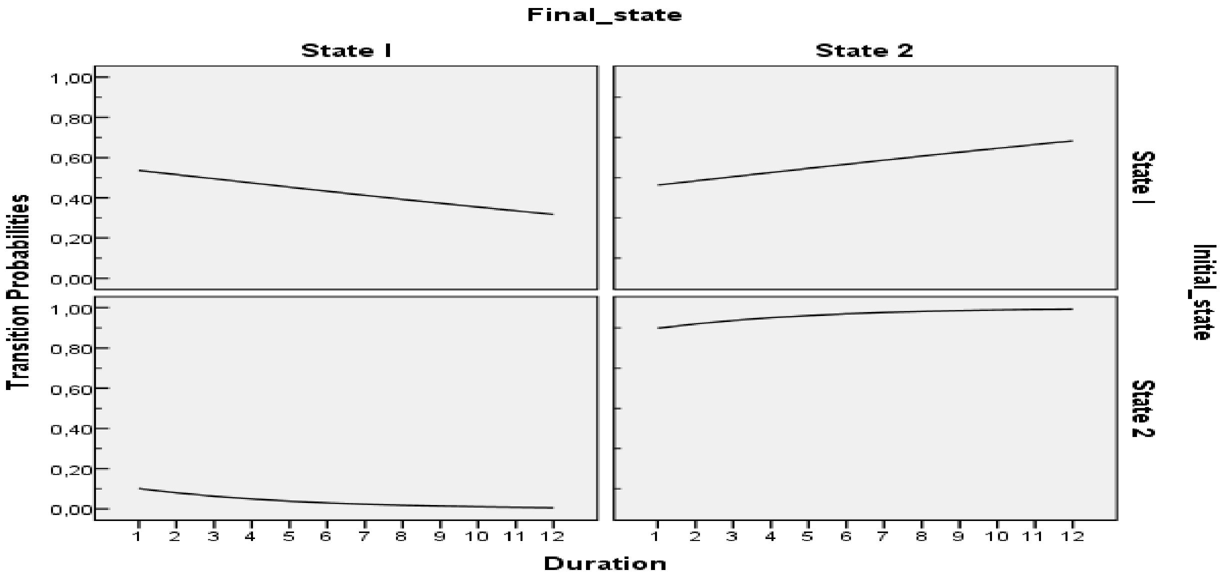

Figure 5. Model (M2) Transition Probabilities

4. MEAN REVERSION IN STOCK RETURNS: LESSONS FROM TWO-STATE DURATION DEPENDENCE MARKOV-SWITCHING MODEL

Using the regime-switching methodology, several studies have classified the fluctuations of stock market into bull and bear states. The bull state is associated with high return and low volatility while the bear state is characterized by low return and high volatility. Our model highlights difference between states both in terms of level, trend, means, variances and transition probabilities. This way of depicting the states is mainly agree with the definition of Chauvet and Potter (2000) according to which the bull market (rep. bear market) corresponds to periods of generally increasing (rep. decreasing) market prices. Thus, we can dichotomise a priori the market conditions using a trend-based scheme. More specially, the decreasing high-return state (state 1) corresponds to the bear state market while the increasing low-return state (state 2) is the bull state market.

In this study, we use the hazard functions in order to examine duration dependence in transition probability. From obtained results, we can again approve cyclical behavior in the TUNINDEX index returns. Furthermore, the existence of increasing (resp. decreasing) hazard function during the bear market (resp. the bull market) means that the probability of staying in the bear market (resp. the bull market) decreases (resp. increases) over time. Duration dependence of market conditions in the conditional return highlights different behaviors of stock returns. In fact, when the stock market is the bearish phase, there is a tendency for the stock returns to decrease in order to reach the equilibrium value. In contrast, during the bullish market, the stock returns increase gradually over time. As a matter of fact, results related to the duration dependence in return volatility show that the decline of volatility over time for two states may indicate slight fluctuations towards a trend; in particular when the stock market is in the state 2.

The implications of all these findings are that returns seem to be reverting to their permanent or trend level in a non random way as the cyclical component dissipates over time. As proposed by Hillebrand (2005), the return process reacts to any deviation from its long term mean. Whether the stock return is below (resp. above) the mean during one period, there is a force that pushes it down (resp. pushes it up) over subsequent periods. Sundry explanations for mean-reverting stock return behavior have been offered in the literature (e.g. Fama and French, 1988; Black, 1990; Cecchetti, Lam and Mark, 1990; Lo and Macklinlay, 1990; Barberis, Shleifer and Vishny, 1998, Hong and Stein, 1999; Gatev and Ross, 2000). Fama and French (1988) put forward that mean reversion arises by virtue of mispricing in an irrational market in which prices take long temporary swings away from fundamental values. In this regard, Balvers, Wu and Gillilard (2000) support the overreaction explanation of the pattern of price continuation followed by mean reversion where positive feedback investors push asset prices away from fundamentals. Mean reversion may be induced by the predictable movement over time in the security risk premia

(Fama and French, 1988). In this respect, Gangopadhyay and Reinganum (1996) show that mean reversion can be ascribed to the CAPM consistent time-varying portfolio risk premia rather than mispricing. According to Cochran (1995), supporting a rational or an irrational explanation for mean reversion has significant implications for the efficient pricing of equity and the opportunity to earn excess profits. Mispricing involves some degree of market efficiency whereas time-varying risk premia is suitable with rational pricing in an efficient market.

In this study, our model provides us some insights to depict the mean reverting stock returns behavior. State 1 is characterized first by the speculative excess caused by the willingness of investors to profit from the short term fluctuations in the stock market and pursue the abnormal returns. As duration increases, stock returns tend to decrease and the speculative bubble tends to disappear. Features of the state 2 seem to indicate the recovery of stock returns towards a trend level. This bounceback characteristic points to a return to fundamentals. Figure 6 illustrates the mean-reverting behavior in the Tunisian stock market.

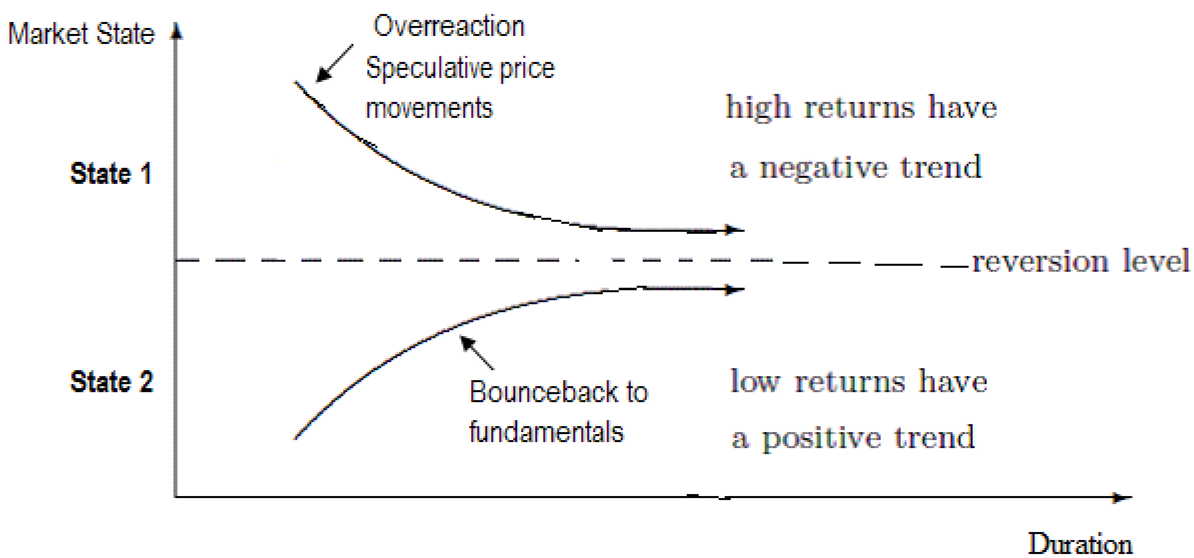

Figure 6. An Attempt Of Depiction The Mean-Reverting Behavior In Tunisian Stock Market

Another salient finding emerges from this study related to the duration dependence in return-risk trade-off. As the two states 1 and 2 persist, any increase in volatility (i.e. volatility shock) has a countercyclical effect on the stock return indicating a break with the cyclical trend of state. This allows us to put forward that the countercyclical return volatility is prompted by large swings of risk premia. Risk premia increases with the length of time spent in the state 1 while it decreases in the state 2. The logic behind this finding is intuitive. The fundamentals are surrounded by fluctuating uncertainty what leads to the investors' changing risk attitudes with the amount of time spent in a particular market state. This results in the asymmetric movements of the risk premia.

All of these findings can only contribute to the understanding of the Tunisian stock market dynamics. Adding an additional state in our duration-dependence Markov-switching model may provide fresh directions into the search process in that we can further highlight the nonlinear dynamics of index returns and clarify the mean-reverting process. Besides, exploring the implications of the issue of mean reversion in the Tunisian stock market on the optimal investment strategy would seem to adduce a useful avenue for future research.

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